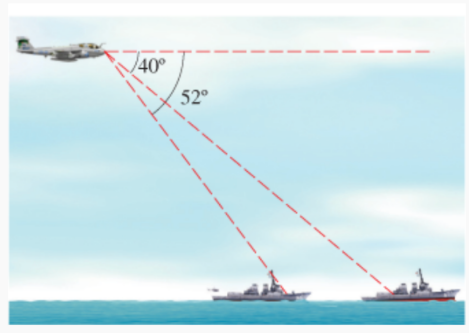
32. **Distance Between Two Ships** A pilot measures the angles of depression to two ships to be 40° and 52° (see the figure). If the pilot is flying at an elevation of $35{,}000$ ft, find the distance between the two ships.



It helps me to draw a larger, neat picture, putting in any angles I can figure out

(1) Using Law of Sines & Right Δs.

40°
52°
12°
35,000
a
b
d
128°
40°

Using law of sines on this triangle,

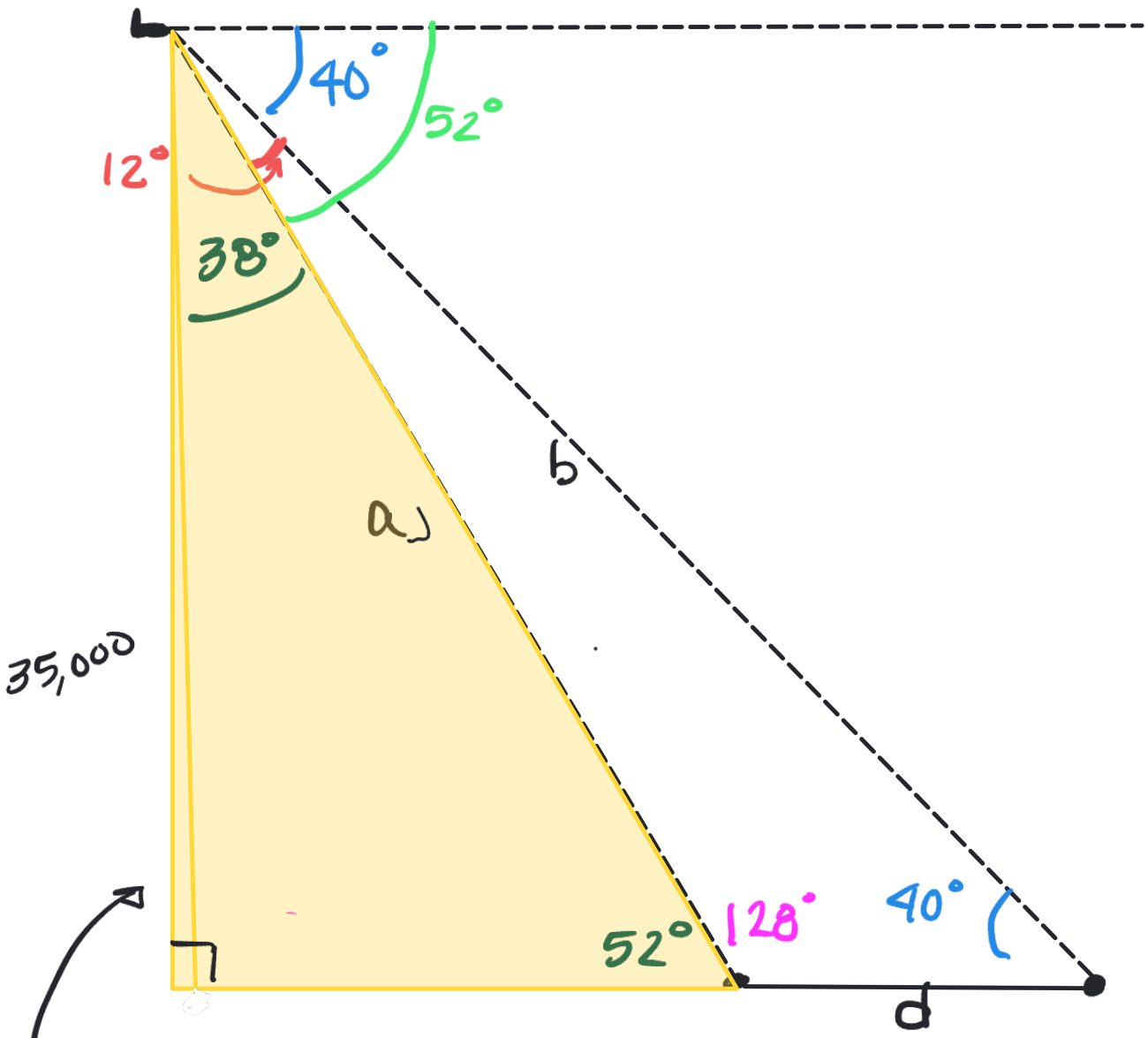
$$\frac{d}{\sin 12^\circ} = \frac{a}{\sin 40^\circ} = \frac{b}{\sin 128^\circ}$$

So if I had a or b I could find d.

There's not enough info in this Δ to get a, b so look for right Δs

$180^\circ - 12^\circ - 40^\circ = 128^\circ$

We haven't yet used this information. Make a right triangle that uses this and will get us a or b



We haven't yet used this information. Make a right triangle that uses this and will get us a or b

Find angles in the right △.

Then $\sin 52° = \frac{35{,}000}{a}$

$a = \frac{35{,}000}{\sin 52°}$

Then from above

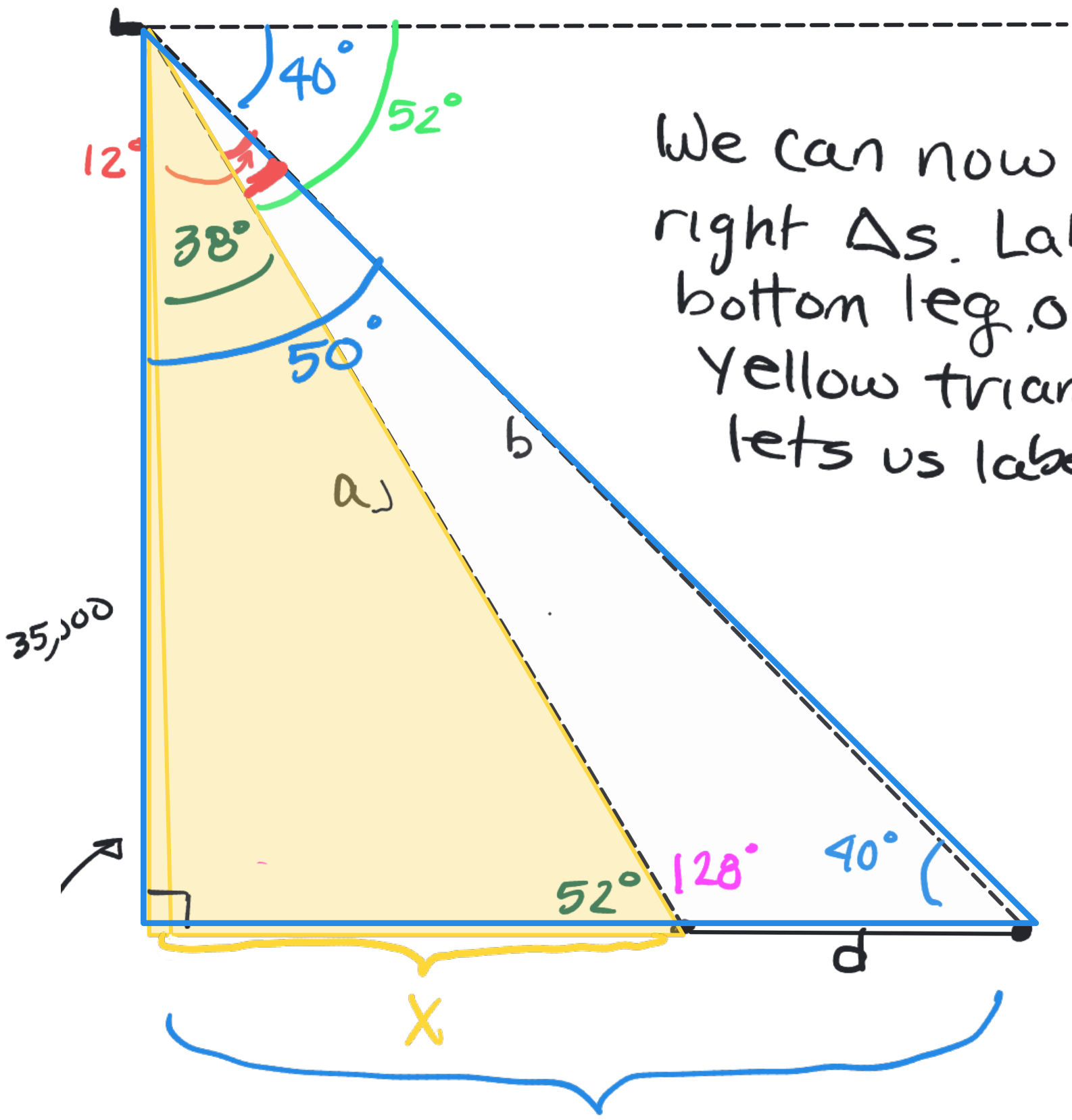
$\frac{d}{\sin 12°} = \frac{a}{\sin 40°} = \frac{b}{\sin 128°}$

This is the exact answer. You don't have to write it like this, but you should be able to use your calc well enough to get

$d = \frac{a}{\sin 40°}\sin 12° = \frac{\frac{35{,}000}{\sin 52°}}{\sin 40°}\sin 12°$ ft

$\approx 14{,}366.379$ ft

② Just Using Right Triangles



We can now see 2 right Δs. Label the bottom leg of the Yellow triangle which lets us label the leg

$\tan 38^\circ = \frac{X}{35{,}000}$

$\tan 50^\circ = \frac{X+d}{35{,}000}$

↳ solve this system

$X = 35{,}000 \tan 38^\circ$

$X+d = 35{,}000 \tan 50^\circ$

$d = 35{,}000 \tan 50^\circ - X$

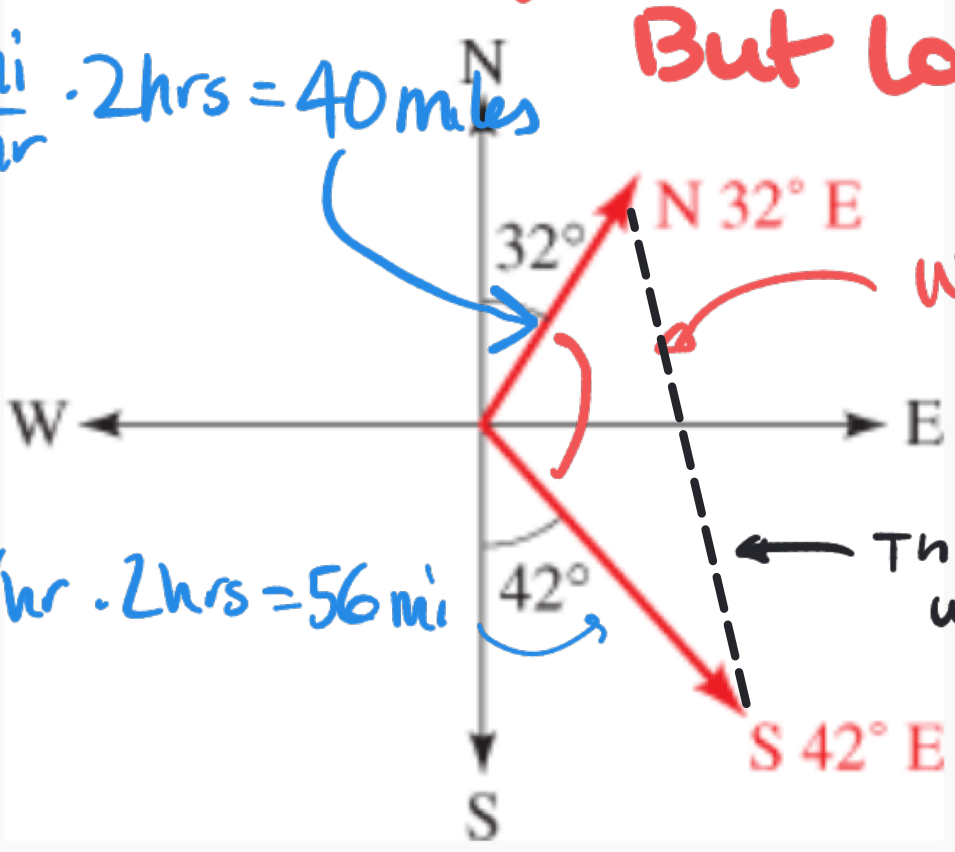
$$d = 35{,}000 \tan 50^\circ - 35{,}000 \tan 38^\circ$$

79. **Distance Between Two Ships** Two ships leave a port at the same time. One travels at 20 mi/h in a direction N 32° E, and the other travels at 28 mi/h in a direction S 42° E (see the figure). How far apart are the two ships after 2 h?

This can also be done with right Δs or Law of Sines - But Law of Cosines is easiest

RT=D

$20\frac{mi}{hr} \cdot 2hrs = 40 miles$

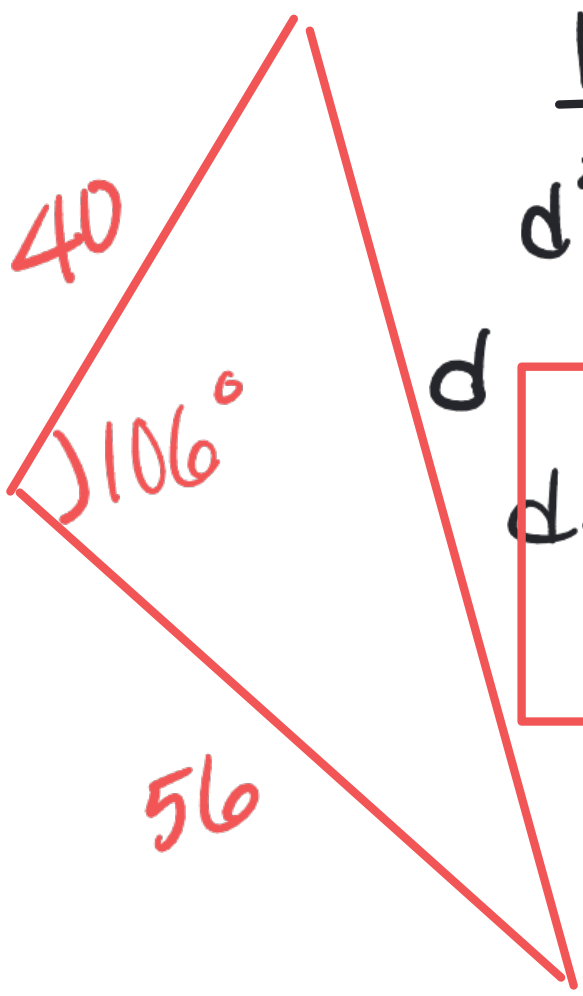


$28 mi/hr \cdot 2hrs = 56 mi$

we can find this angle by $180° - 32° - 42° = 106°$

← This is the distance we want.

We can make a triangle with this info.



Law of Cosines

$d^2 = 40^2 + 56^2 - 2(40)(56)\cos 106°$

$d = \sqrt{4736 - 4480\cos 106°}$

≈ 77.27 miles